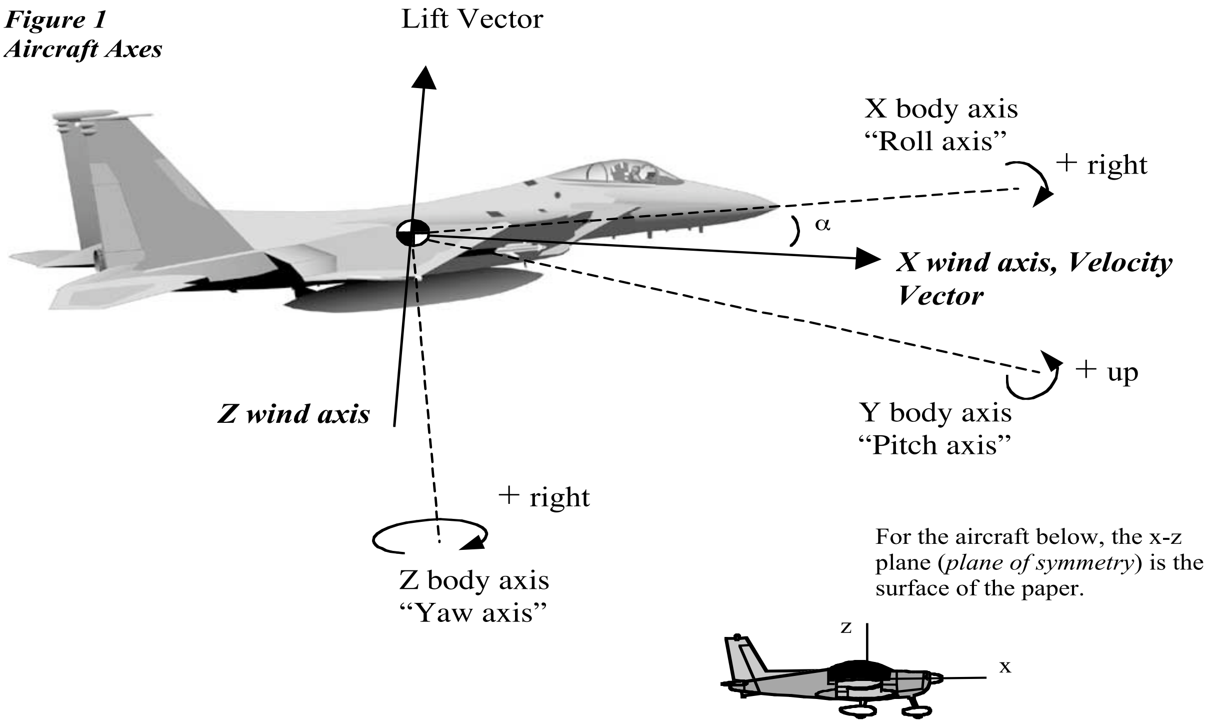
Flightlab Ground School
1. Axes and Derivatives

Figure 1
Aircraft Axes



For the aircraft below, the x-z plane (*plane of symmetry*) is the surface of the paper.

Introduction

If you didn't much care for symbols, formulas, and coefficients back in primary ground school, the following may raise warning signals. Ignore them and don't be a wimp. You'll want to understand the axis system and also to take a look at the tables of aerodynamic derivatives (which we'll review in person, as well). The derivatives break aircraft behavior down to cause and effect, giving the engineers lots to calculate and giving us the terms needed to evaluate aircraft in an informed, qualitative way—a way that links the demands of airmanship to the specific personalities of our machines.

Aircraft Axes

The dashed lines in Figure 1 describe an aircraft's x-y-z *fixed body axes*, emanating from the center of gravity. This system, with the mutually perpendicular axes in fixed reference to the aircraft, is the one most pilots recognize. The exact alignment is a bit arbitrary. Boeing sets the x-axis parallel to the floorboards in its aircraft.

The geometrical plane that intersects both the x and z body axes is called the *plane of symmetry*, since a standard aircraft layout is symmetrical left and right (Figure 1, bottom).

There are alternative axis systems (zero-lift body axis, stability axis, for example). For pilots, the *wind axis* system is the most useful, because it best helps in visualizing how aircraft actually behave.

The wind axis system sets the x-axis in alignment with the aircraft's *velocity vector,* which points in the direction in which the aircraft is actually moving. Usually the velocity vector/wind axis lies on the aircraft's plane of symmetry, but not always. If the aircraft is in a sideslip, the velocity vector moves off the plane to some sideslip angle, β ("beta"), as Figure 2 illustrates.

The velocity vector also changes direction when aircraft angle of attack, α ("alpha"), changes.

The velocity vector is projected onto the x-z plane of symmetry for measuring α, and onto the x-y plane for measuring β. Thus it contains both α and β, as the bottom of Figure 2 shows.

Both the y and z wind axes remain perpendicular to the x wind axis (and to one another). So, as the velocity vector changes direction, these axes change orientation, as well. Thus they're carried along by the aircraft, but not "fixed."

Here's the essence of why the velocity vector is important to pilots: Much of aircraft response is pinned to it, both during normal flight and in unusual attitudes.

Laterally and directionally stable aircraft normally tend to roll away from, but yaw toward, the velocity vector when the vector is off the plane of symmetry. Unstable aircraft lack these instincts, or lack them in proper combination.

In addition, ***a trimmed, longitudinally stable aircraft tends to hold the velocity vector at a constant angle of attack, unless commanded otherwise***. An unstable aircraft does not.

Aerodynamically stable aircraft tend to roll, pitch, and yaw around their respective wind axes—not around their fixed body axes, as most pilots are taught. The picture becomes more complicated when those axes then begin to change their direction in space,[1] but a simplified notion of wind axis rotation is often helpful in visualizing maneuvering flight.

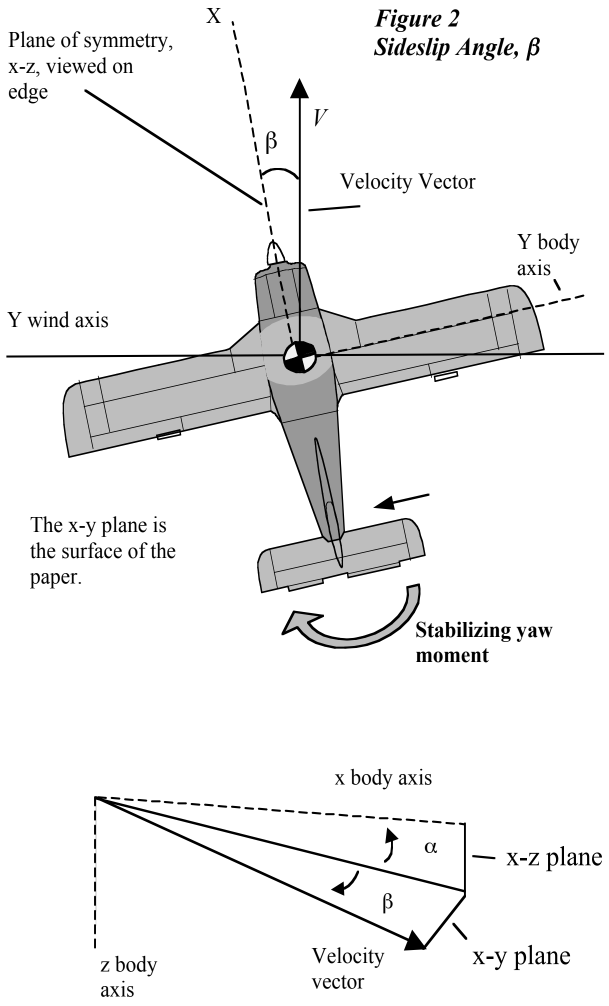


Figure 2
Sideslip Angle, β

There's another axis system, based on the aircraft's distribution of mass: the *inertia* (or *principal*) axis system. The moments of inertia about the three, mutually perpendicular, principal axes determine how quickly rates of roll, pitch and yaw can change around the aircraft's center of gravity. (For example, an aircraft with tip tanks has more x-axis roll inertia when the tanks are full than when empty, and for a given airspeed, altitude, and aileron deflection will take longer to achieve a roll rate. It will also take longer to stop rolling.) The principal axes are the lines around which mass is symmetrically arranged. They may not always be shown as coincident with the aircraft fixed body axes—although, because aircraft are essentially symmetrical, they're often close enough to be considered as such. Differences in moments of inertia around each axis can lead to various coupling effects. We'll leave the details until our discussion of spins.

---

[1] Kalviste, J., "Spherical Mapping and Analysis of Aircraft Angles for Maneuvering Flight," AIAA-86-2283.

Lift Vector

A directionally stable aircraft returns the velocity vector to the plane of symmetry if the vector becomes displaced to some sideslip angle, β (as the "stabilizing yaw moment" is doing in Figure 2). ***In coordinated flight, the velocity vector lies on the plane of symmetry*, *as does the lift vector.***

As illustrated in Figure 1, the lift vector is the upward projection of the z wind axis. Since lift is perpendicular to the air stream generated by the aircraft's velocity, it makes sense to think of its vector in wind axis terms. Fighter pilots talk cryptically of keeping the lift vector on the bogey, while an instructor might direct an inverted-attitude recovery by saying "roll the lift vector toward the sky." They generally mean a fixed vector perpendicular to the wingspan—bolted on, figuratively speaking. That's sufficient and appropriate most of the time. The direction relative to the horizon of the lift vector so defined has a profound effect on an aircraft's maneuvering performance (see the ground school text "Maneuvering Loads, High-G Maneuvers"), but it's also possible to consider the lift vector as free to rotate around the x-axis, as it does in uncoordinated flight. For example, if a pilot uses top rudder (fuselage lift) to keep the nose up during a steep bank, the lift vector will tilt toward the high wing. Sometimes it's useful to think of the lift vector as staying oriented in space while the aircraft rotates beneath it, as it does, essentially at least, during a properly flown "slow" roll. Halfway through the slow roll, when the pilot pushes on the stick and the aircraft is producing lift inverted, the lift vector points heavenward, as it does normally, but now poking out the belly. At each knife-edge, when the wings are unloaded and the pilot presses top rudder so that the fuselage is used briefly for lift, the vector still points heavenward, but out the side. We'll refer to a fixed or free lift vector, as the situation requires.

Signs, Moments, Symbols

In the sign system used with the axis notation, positive values are in the direction shown by the curved arrows in Figure 1, negative values are opposite. For example, when you pull the stick back and add left aileron, you're generating a positive pitching moment and a negative rolling moment (therefore a positive pitch rate and angle, and a negative roll rate and angle). The signs are not related to the aircraft's attitude relative to the earth or to the pull of gravity.

A moment is a force producing rotation around an axis. An aerodynamic moment is the product of a force acting on a surface—say the center of pressure of a vertical stabilizer with a deflected rudder—times the perpendicular distance from that surface to the respective axis—the z-axis for a deflected rudder. *When an aircraft is in equilibrium about an axis, all the positive and negative moments around the axis sum to zero.*

We'll often break down our training aircraft's behavior into its x-y-z, roll-pitch-yaw components. Changes in aircraft attitude or angular velocity (rotation rate) are the result of

Axis	Moment Applied	Angular Velocity	Angular position	Moment of Inertia	Control Deflection
x	l	Roll rate, p	Roll angle ϕ (phi)	I_{xx} Roll Inertia	Aileron (δa)
y	m	Pitch rate, q	Pitch angle θ (theta)	I_{yy} Pitch Inertia	Elevator (δe)
z	n	Yaw rate, r	Yaw angle ψ (psi)	I_{zz} Yaw Inertia	Rudder (δr)

changes in moments applied around each axis. You already know the primary moments (ailerons produce rolling moments, elevators pitching moments, rudders yawing moments), but there's a further collection of direct and cross-coupled moments essential to aircraft control and often complicit in unusual attitudes. We'll talk about them on the ground and observe them in flight.

For reference, the table above shows notations used for moments, angular velocities, angular positions, moments of inertia, and control deflections about each aircraft axis. You don't need to memorize any of this for our course, but you might find it useful for future technical reading. Notice the preference for arranging things by alphabetical order. Thus the letters don't always mean what your mnemonically inclined brain would like them to mean ("r" doesn't stand for roll rate; "p" doesn't stand for pitch rate, and, while "L" stands for lift, a lowercase "l" stands for roll moment).

Stability and Control Derivatives

Moments about the axes drive aircraft attitude. Stability is the tendency of an aircraft to generate the aerodynamic moments necessary to return it to its original equilibrium, when disturbed. During unusual attitudes, if an aircraft is left to its hands-off free response, those same moments can become destabilizing. At high bank angles, for example, directional stability (a yawing moment) causes the nose to descend below the horizon and speed to increase. When an aircraft is inverted, longitudinal stability (a pitching moment) causes the nose to fall below the horizon, as well. And at angles of attack past stall, rolling moments that would ordinarily damp out can instead produce autorotation and spin departure.

In normal maneuvering in a stable aircraft, a pilot uses the controls to overcome the aircraft's stabilizing moments and to establish a new equilibrium, at least temporarily. This may be easy or not so easy, depending on the degree of inherent stability and the availability of control power to do the job

Stability and control are measured in terms of derivatives—the rate of change of one variable with change in another. During our flights, especially early on, we're going to see how the rates of change of moments in pitch, roll, and yaw can vary with angle of attack, sideslip angle, the presence of aerodynamic and/or inertial couples, control deflections, and with airspeed. The derivatives in the tables that follow form the basic vocabulary of cause and effect that we'll apply in analyzing departure modes and in learning to recover from unusual attitudes. Some will be new to you (as perhaps all the symbols), and some you'll remember, at least in general terms, from the days of primary ground school. Don't worry about learning the symbols. We'll refer to things by name.

Initially, you might want to review the descriptions—which are by necessity condensed—and then refer to the Flightlab Ground School texts for more explanation. We'll also brief the material before flying. Don't feel responsible for immediately understanding all of the bulleted items. You'll get there in stages. Top priority goes to acquiring new flying skills.

A note on signs: The derivatives carry signs that might be confusing at first. A negative (–) sign doesn't indicate the lack of stability, but rather helps determine the direction of response. Review the sign system used with the axis notation in Figure 1. Then, in the derivative table, note for example that $C_{l\beta}$, the lateral stability derivative, carries a negative sign. When a laterally stable aircraft slips to the right (positive direction) it will roll to the left (negative direction). Algebraically, a negative (the derivative) times a positive (sideslip direction) equals a negative (roll direction). If the aircraft slips to the left, it will roll to the right, since a negative derivative times a negative direction equals a positive. For us, the signs will come in handy when analyzing spins, where they simplify the perplexity inherent in understanding a flight regime where an input in one axis can produce an output in another.

The flow chart at the end of this section shows a related way of describing the basics of aircraft response.

Selected Aerodynamic Derivatives for Roll

Aerodynamic Stability Derivative Symbol	Name	Description
$-C_{l_{\beta}}$ C = coefficient l = roll moment β = sideslip angle	Rolling moment due to sideslip. (Lateral stability produced by dihedral effect)	Aircraft rolls away from the direction of sideslip. Main causes are geometrical dihedral and/or wing sweep, and fuselage-induced airflow changes that place the wings at different angles of attack. • Roll due to sideslip is proportional to sideslip angle, β, and to the coefficient of lift, C_L, up to the stall, but may vary afterwards. • Roll rate commanded by aileron/spoilers is affected by sideslip angle and direction. • Wingtip washout, and/or flap deployment, reduce $C_{l_{\beta}}$. • Depends on wing position relative to fuselage. • Decreased by wing taper and low aspect ratio (wingspan^2/wing area)
$-C_{l_p}$ l = roll moment p = roll rate	Rolling moment due to roll rate. (Roll damping)	As an aircraft rolls in response to a disturbance, the angle of attack increases on the down-going wing and decreases on the up-going wing. The resulting change in lift produces an opposing rolling moment. The aircraft stops rolling. If the pilot holds aileron deflection, roll damping moment builds until it's equal to the opposing moment produced by the aileron deflection. Roll rate then becomes constant. • Roll damping disappears on wing sections at stall; autorotation is the reversal of roll damping. • Damping increases with the slope of the C_L curve. • Reduced by low aspect ratios and/or wing taper. • Roll damping decreases with altitude.
$+C_{l_r}$ l = roll moment r = yaw rate	Rolling moment due to yaw rate.	Yaw rate causes airflow velocity to increase on the advancing wing and decrease on the retreating wing, causing a spanwise change in lift and a rolling moment. • The effect follows the lift curve, becoming greatest at C_{Lmax} and then falling off after the stall. (C_{l_r} = approx. $C_L/4$). • Rolling moment due to yaw rate contributes to spiral instability and to spin departure. • When entering a sideslip, rolling moments due to the temporary yaw rate and the growing sideslip angle are additive. • Wingtip washout, and/or flap deployment, reduces C_{l_r} . • Little affected by wing position on fuselage. • Increases with aspect ratio, decreases with wing taper. • Varies with the square of the difference in tip speed (since lift varies with V^2).

Selected Aerodynamic Derivatives for Yaw

Aerodynamic Stability Derivative Symbol	Name	Description
$+C_{n_\beta}$ n = yaw moment β = sideslip angle	Yawing moment due to sideslip. (Directional stability)	Also known as weathercock stability. Aircraft yaws toward the direction of sideslip to align the longitudinal, x-axis with the relative wind. • The fuselage alone is usually destabilizing; principal stability contribution comes from vertical tail, although swept wings are stabilizing, an effect that increases with C_L. • Spiral instability occurs when directional stability is high and lateral stability is low. • Low directional stability and high lateral stability promotes Dutch roll.
$-C_{n_p}$ n = yaw moment p = roll rate	Yawing moment due to roll rate.	The induced change in angle of attack on a rolling wing causes the lift vector to tilt back on the wing going up, and forward on the wing going down. This adds components of thrust and drag, which produce a yawing moment opposite the direction of roll (similar to adverse aileron yaw). • Increases with aspect ratio, roll rate. • Increases with C_L. • Wingtip washout, and/or flap deployment, reduces C_{n_p}. • Largely independent of taper. • (C_{n_p} = approx. $C_L/8$). • Reverses effect when the wing goes into autorotation.
$-C_{n_r}$ n = yaw moment r = yaw rate	Yawing moment due to yaw rate. (Yaw damping)	When an aircraft has a yaw rate, opposing aerodynamic damping forces build up ahead and behind the center of gravity. • Main contribution comes from the vertical tail, but the forward fuselage can also contribute (unlike C_{n_β}, in which the fuselage forward of the wing is destabilizing). • Wings also contribute, since the advancing wing produces more induced and profile drag than the retreating wing. • Wing contribution to yaw damping increases with angle of attack; the tail's contribution may decrease due to disrupted airflow at high α. • Yaw damping decreases with altitude.

Rudder/Aileron Cross Derivatives

Control Derivative Symbol	Name	Description
$C_{l_{\delta r}}$ l = roll moment δ = deflection r = rudder	Rolling moment due to rudder deflection.	A roll moment is produced if the lift generated by rudder deflection acts at a point above the roll axis. Right rudder, for example, produces a left rolling moment. This can become apparent in aircraft without dihedral effect. • Diminishes as angle of attack increases.
$C_{n_{\delta a}}$ n = yaw moment δ = deflection a = aileron	Yawing moment due to aileron deflection. (Adverse yaw)	An aileron deflected down creates more *induced* drag than the opposite aileron deflected up. The result is a yawing moment opposite the direction of bank. *Profile* drag increases on both wings when the ailerons are deflected, the difference depending on aileron design. • Adverse yaw increases with wing angle of attack, because drag rises faster than lift at high α. • Spoilers for roll control can produce proverse yaw. • Differential ailerons or Frise ailerons counteract adverse yaw with opposing drag—although their primary function is to lower aileron control force.

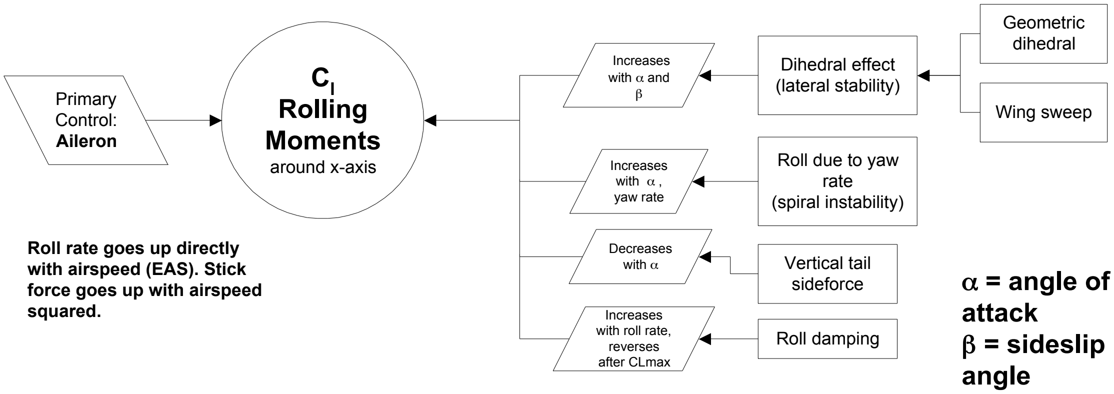
Pitch Damping

Aerodynamic Stability Derivative Symbol	Name	Description
C_{m_q} m = pitching moment q = pitch rate	Pitching moment due to pitch rate. (Pitch damping)	When aircraft pitches up or down, the motion of the horizontal stabilizer causes a change in the stabilizer's angle of attack, which generates an opposing, or damping, pitching moment. • Pitch-damping moment increases with pitch rate (and thus with g load). • Pitching moment due to pitch rate affects short-period response and stick force per g in pull-ups and turns. • Pitch damping decreases with altitude. • Pitch damping increases with increased distance between the horizontal stabilizer and the aircraft c.g.

Axes and Derivatives

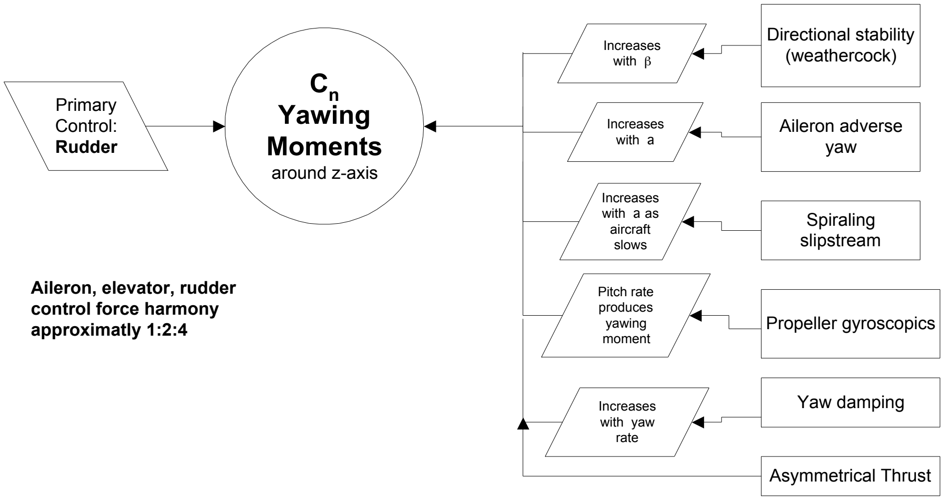
Control Deflection, Pilot-Generated Moments

Aerodynamic, Aircraft-Generated Moments



Roll rate goes up directly with airspeed (EAS). Stick force goes up with airspeed squared.

α = angle of attack
β = sideslip angle



Aileron, elevator, rudder control force harmony approximatly 1:2:4